IS A €10 TRILLION EUROPEAN CLIMATE INVESTMENT INITIATIVE FISCALLY SUSTAINABLE?
APPENDIX

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Technical appendix

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Contents

1 Introduction **1**

2 Data **1**

3 The modelling and identification approach **2**

3.1 Identification approach . . . 3

3.2 Step Indicators . . . 3

4 Investment spending policy shocks **4**

5 Semi-permanent structural IRFs (SP-SIRF) **4**

6 Marginal effects and multipliers **5**

7 Coordinated and uncoordinated fiscal policy **5**

8 A law of motion for government debt **6**

1 Introduction

This appendix outlines the VAR model and the data which was used to produce the results of the main document. Most importantly it provides an exact definition of the key concepts such as the **long run multipliers (LRMs)**, the **cumulative impulse response functions (CIRFs)** and the **semi-permanent impulse response functions (SPIRFs)**. The appendix also outlines how long run multipliers were averaged across EU27 member states and provides a brief derivation of the standard law of motion for government debt used in section 2 of the main report.

2 Data

The results are based on quarterly data for government investment spending $(GINV)$[1], output (GDP)[2] and the stock of government debt $(GDEBT)$[3]. All three series have been obtained

[1] Gross fixed capital formation for the general government sector from Eurostat table *gov_10q_ggnfa*

[2] From Eurostat table *namq_10_gdp* in chain linked volumes.

[3] General government consolidated gross debt from Eurostat table *gov_10q_ggdebt*

from Eurostat, where possible already seasonally and calendar adjusted, and deflated with the implicit GDP deflator. The $GINV$ and $GDEBT$ series were seasonally and calendar adjusted with Python's statsmodels x13 package. The quarterly GDP and $GINV$ series were transformed into annualized rates (i.e. multiplied by 4) to achieve easier comparison with annual data. All three series are plotted for the EU27 below.

Figure 1: EU27 real GINV and real GDP (chain linked volumes)

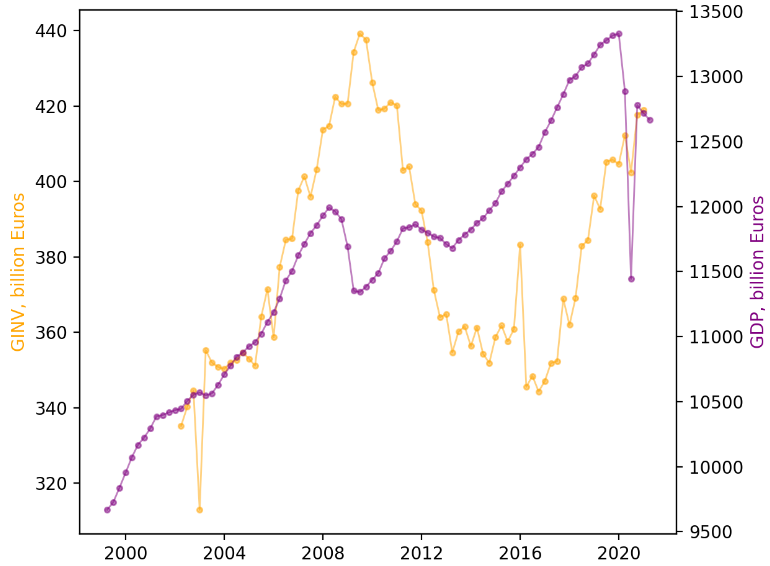
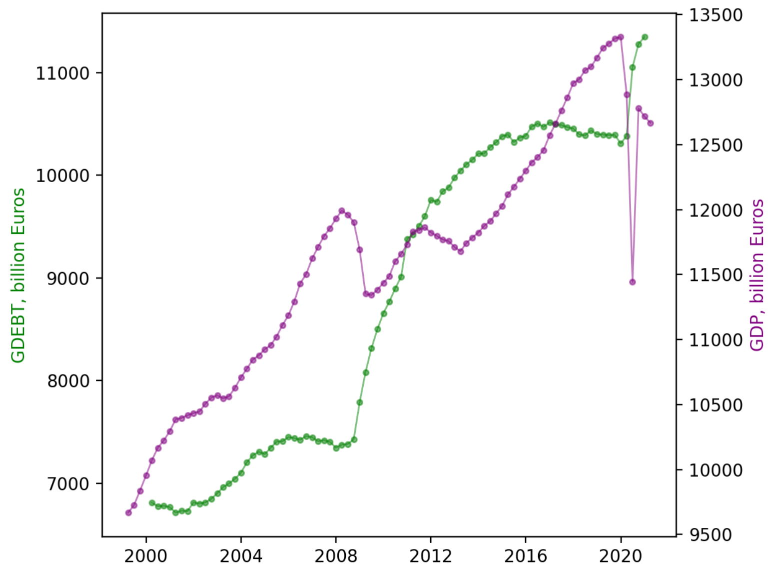


Figure 2: EU27 real GDEBT and real GDP (chain linked volumes)



3 The modelling and identification approach

We are working with a structural VAR model of the following form:

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + m_0 + m_1 t + \sum_{i=1}^{s} m_{2,i} S_i + \omega_t \quad (1)$$

where p is the lag length, y_t is a vector of K endogenous variables of the dimensions $K \times 1$, the B matrices are $K \times K$ coefficient matrics, m_0 is a $K \times 1$ vector of constants, m_1 is a $K \times 1$ vector of time trends and $m_{2,i}$ are a $K \times K$ coefficient matrices for s step indicators represented by the $K \times 1$ vectors S_i. We use two models, in the first $K = 2$ and $y_t = [g_t,\ x_t]'$ and in the second $K = 3$ and $y_t = [g_t,\ x_t,\ d_t]'$ where g_t is government investment spending, x_t is GDP and d_t is the stock of government debt. All variables are deflated by the GDP deflator and in logarithms. We will discuss identification first and then introduce the step indicator approach.

3.1 Identification approach

The contemporaneous effects matrix B_0 is of a lower triangular form (for the three variable case):

$$B_0 = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \tag{2}$$

Our identification approach relies on one main assumption which is that government investment does not react within the period to either GDP or government debt. Which means we order the variables within y_t in the following form:

$$y_t = \begin{bmatrix} g_t \\ y_{2,t} \\ y_{3,t} \end{bmatrix} \tag{3}$$

Since we are only interested in the causal effects of government investment spending (g_t) but do not attempt to identify other demand or supply shocks or shocks to government debt, the ordering of the remaining variables in the system does not affect the government investment impulse response functions.

3.2 Step Indicators

The significant slowdown of the trend growth rate after the 2009 financial crises and the 2012 Euro crisis, is not just a temporary deviation from an otherwise unchanged long term trend (captured by the included time trends). This phenomenon has received substantial attention in the literature where it was discussed under the labels of *secular stagnation*, *hysteresis* and *austerity*. In order to strike a balance between keeping the model relatively simple and trackable while being required to take this crisis of historic proportion into account, we model these crises as exogenous events by incorporating *step-indicator saturation* as discussed by Castle et al. (2015). The idea of step indicator saturation is to saturate the model with step indicators S_i for each quarter t where S_t is equal to 1 from the first quarter up to quarter t and zero afterwards:

$$S_t = (\underbrace{1, \ldots, 1}_{\text{t times}}, \underbrace{0, \ldots, 0}_{\text{T-t times}}) \tag{4}$$

This means step indicator S_t allows for a permanent shift (i.e. a step) in the time series. The estimated coefficient matrix $m_{2,s}$ determines the sign and size of this shift for each of the K endogenous variables. We estimate the model by including 1 step indicator and then re-estimate it with the next step indicator. We repeat this process for all $T - 1$ step indicators. The finally selected model is estimated with those step indicators which are statistically significant at the 1% level and up to a maximum of 7 step indicators (10% of the sample).

4 Investment spending policy shocks

The MA representation of the model is given by:

$$y_t - y_t^p = \sum_{i=0}^{\infty} \phi_i u_{t-i}$$

$$y_t - y_t^p = \sum_{i=0}^{\infty} \phi_i B_0^{-1} B_0 u_{t-i}$$

$$y_t - y_t^p = \sum_{i=0}^{\infty} \Theta_i \omega_{t-i} \tag{5}$$

where y_t^p is the particular solution or the steady state of the system, $\Theta_i = \phi_i B_0^{-1}$ and $\phi_i = J\boldsymbol{A}^i J'$ where $\boldsymbol{A}$ is the companion matrix of the VAR(p) process (Kilian & Lütkepohl 2017, p. 25). The Θ_i matrices are $K \times K$ with elements $\theta_{jk,i}$ where j indicates the row and k the column. This means we have an MA representation in the structural shocks ω_t rather than the reduced form errors u_t. The structural impulse response function (SIRF) to a one off (temporary) structural shock is given as:

$$\frac{\partial y_{j,t}}{\partial \omega_{k,0}} = \text{SIRF}_{jk,t} = \theta_{jk,t} \tag{6}$$

Therefore $\theta_{jk,t}$ gives the deviation from the steady state or particular solution of variable j, t periods after a structural shock ω hit variable k in period 0. We can calculate the cumulative structural IRF (C-SIRF) at horizon t as:

$$\text{C-SIRF}_t = \sum_{i=0}^{t} \theta_{jk,i} \tag{7}$$

For an infinite horizon C-SIRF$_\infty$ becomes the effect of a permanent change in the intercept:

$$\frac{\partial y_{j,t}^p}{\partial m_{k,0}} = \sum_{i=0}^{\infty} \theta_{jk,i} = \text{C-SIRF}_\infty \tag{8}$$

Thus we can use the C-SIRF to compute the effect of a permanent exogenous change in investment spending ($m_{g,0}$), for example due to a policy change, on the steady state (particular solution) of the system. Overall the SIRF enables us to trace the effect of a one-off or temporary shock through the system and the C-SIRF enables us to trace the effect of a permanent shock through the system. What is less straightforward is how to trace a shock through the system which occurs for more than 1 period but is not permanent. In the next section we will combine C-SIRFs to do that.

5 Semi-permanent structural IRFs (SP-SIRF)

Let's say we want to track the effect of a shock to variable k on variable j and let's assume this shock lasts for a specific number (l) of periods. First, we can rely on the C-SIRFs to calcualte the deviation from the steady up to period l. From period $l+1$ onward the shock recedes and the endogenous adjustment back to the steady state begins. We can track the full adjustment using the following formula:

$$y_{jk,t} - y_{jk,t}^p = \sum_{j=t}^{t-l} \theta_{jk,j} = \sum_{j=0}^{t} \theta_{jk,j} - \sum_{j=0}^{t-l-1} \theta_{jk,j} = \text{C-SIRF}_{jk,t} - \text{C-SIRF}_{jk,t-l-1} = \text{SP-SIRF}_{jk,t} \tag{9}$$

and we will call it a semi-permanent structural impulse response function (SP-SIRF). The formula for calculating SP-SIRFs can be derived from using the MA representation $y_t - y_t^p = \sum_{i=0}^{\infty} \Theta_i \omega_{t-i}$ and plugging in a specific sequence of shocks such as $(\dots, 0, 0, 0, \omega_0, \omega_1, \dots, \omega_l, 0, 0, \dots)$ and collecting terms.

6 Marginal effects and multipliers

In the example of fiscal policy, we are interested in linking the size of the impulse and the endogenous response of the fiscal instrument to the size of the output response triggered by the fiscal impulse. Fiscal multipliers are a way of achieving exactly that by condensing this relationship into a single number. If the data vector y_t consists of or contains time series in logarithms, the IRFs represent elasticities or percentage deviations from the steady state. In this context prior to calculating fiscal multipliers, these elasticities need to be transformed into marginal effects. IRFs expressed in marginal effects (ME) represent the deviations from the steady state of variable j not in percentages but in levels. For this purpose it is common in the fiscal multiplier literature Ramey (2019), Gechert et al. (2021) to multiply the SIRFs with the sample mean of the underlying response variable j. Alternatively the sample end, or start or any other period could be used instead of the sample average:

$$ME_{jk,t} = \bar{y}_j \theta_{jk,t} \tag{10}$$

In our application the marginal effect of a one standard deviation shock to government investment spending (g, the k-varible) on GDP (x, the j variable) is:

$$ME_{xg,t} = \bar{x} \theta_{xg,t}$$

$ME_{xg,t}$ is given in the units of measurement of x which are billion Euros. The cumulative fiscal multiplier (CFM) at horizon t is then given as:

$$CFM_t = \frac{\sum_{i=0}^{t} ME_{xg,i}}{\sum_{i=0}^{t} ME_{gg,i}} \tag{11}$$

which is the ratio between the total deviation of GDP from the steady state and the total deviations of the fiscal variable from steady state in response to a permanent increase in the fiscal variable.

7 Coordinated and uncoordinated fiscal policy

We have aggregate data for the EU27 and in addition data for all 27 member states individually. This allows us to compare the effectiveness of fiscal policy between periods of coordinated (or simultaneous) expansions and periods of isolated expansions. For this purpose we calculate the cumulative fiscal multiplier based on aggregate EU27 data and label it as:

$$CFM_t^{EU27} \tag{12}$$

This multiplier can be interpreted as a measure of fiscal policy effectiveness based on simultaneous fiscal expansions in the EU27. We can compare that with country specific fiscal multipliers based on individual country data:

$$CFM_t^c$$

where $c = (\text{AT}, \dots, \text{SK})$. CFM_t^c represents the effectiveness of fiscal policy in country c based on an isolated fiscal expansion in country c. In order to compare the effects of isolated or uncoordinated fiscal expansions we summarise the 27 individual multipliers in order to easily compare them to CFM_t^{EU27}. The first way of summarising them is to calculate a GDP weighted average to take the different sizes of the member states' economies into account:

$$CFM_t^{AV1} = \sum_{c=AT}^{SK} \frac{x_t^c}{x_t^{EU27}} CFM_t^c \tag{13}$$

When calculating CFM_t^{AV1} we define $x_t^{EU27} = \sum_{c=AT}^{SK} x_t^c$ because it will slightly deviate from x_t^{27} due to the non-summability of national accounts data in chained volume indeces. The second way of summarising the individual responses is to aggregate the marginal effects:

$$CFM_t^{AV2} = \frac{\sum_{c=AT}^{SK} \sum_{i=0}^{t} ME_{xg,i}^c}{\sum_{c=AT}^{SK} \sum_{i=0}^{t} ME_{gg,i}^c} \tag{14}$$

The first approach is more intuitive because it is a simple (GDP-weighted) average across 20 individual fiscal multipliers. The second approach is more accurate because it correctly adds up the deviations from the particular solutions for each country and only as the last step takes the ratio. We report both approaches, the results are qualitatively similar.

8 A law of motion for government debt

In order to derive a simple law of motion for government debt we start by defining government debt $B(t)$ as well as real GDP $Y(t)$ as functions of time. Further we define the debt to GDP ratio as $b = \frac{B(t)}{Y(t)}$. Taking the derivative of b with respect to time by applying the chain rule yields:

$$\frac{db}{dt} = \dot{b} = \frac{\dot{B}Y - B\dot{Y}}{Y^2} = \frac{\dot{B}}{Y} - b\frac{\dot{Y}}{Y} = \frac{\dot{B}}{Y} - b\hat{y} \tag{15}$$

variables with dots represents derivatives of these variables with respect to time and $\hat{y}$ is the growth rate of real GDP. Next we will make use of the following accounting identity:

$$\dot{B} = E - T + iB \tag{16}$$

where E-T is the primary deficit and i is the current interest rate on government debt. It states that the change in government debt $\dot{B}$, is equal to the primary budget deficit plus the interest rate payments on the outstanding public liabilities. Combining equations (15) and (16) yields:

$$\dot{b} = (e - t) + (i - \hat{y})b \tag{17}$$

where g-t is the primary deficit in percent of GDP and $(i - \hat{y})$ is the difference between the real interest rate and real output growth rate[4]. The stability of this differential equation is given iff:

$$\frac{\partial \dot{b}}{\partial b} = (i - \hat{y}) < 0 \tag{18}$$

which means if the growth rate is above the interest rate. Then the steady state d^* is given by:

$$b^* = \frac{t - e}{i - \hat{y}} \tag{19}$$

As discussed in the main text, this way of looking at public sector dynamics ignores any feedback from government spending E on output Y itself. Thus this law of motion is at best a crude long run rule of thumb if one is prepared to assume that the growth of the economy is driven by factors such as demographis or technological change with themselves are not affected by government spending.

[4]The discrete time version of this equation becomes $\Delta b_t = (e - t) + (i - g)b_t$ where $g = \frac{\Delta Y}{Y}$ is the growth rate of output. This is equivalent to equation (1) in the main text.

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